The next three problems deal with the part "integers, division, and modular arithmetic" of the course *Introduction to Mathematics*. For each of them you obtain at most 6 points; together with 2 extra points you get for free, this means that you can score between 2 and 20 points for this part of the exam. 28 November 2013.

(1) [6 points] Suppose that n and m are integers and that at least one of them is nonzero. Prove that

$$\gcd(n+m, 2n) = \gcd(n+m, 2m).$$

(2) ([2 points]) Is $42 \bmod 101 \in \mathbb{Z}/(101)$ a unit?
Find an integer x such that $42x \equiv 47 \bmod 101$ ([4 points]).

(3) This problem discusses the integer $n := 28^{112013} - 1288$.
   (a) [1 + 1 points] Explain why n is divisible by 8 and by 7.
   (b) [1 + 1 points] Explain why n is divisible by 5 and by 10.
   (c) [2 points] Prove that $m|n$ for every $m \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.